



K230 2303

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/  
Improvement) Examination, November 2023

(2019-2021 Admissions)

CORE COURSE IN MATHEMATICS

5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

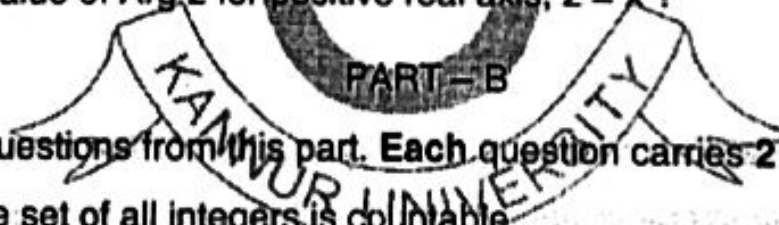
Time : 3 Hours

Max. Marks : 48



Answer any 4 questions from this part. Each question carries 1 mark. (4x1=4)

1. Give example for a denumerable set.
2. If  $\alpha, \beta, \gamma$  are the root of the equation  $f(x) = 0$ , then the equation whose roots are  $-\alpha, -\beta, -\gamma$  is \_\_\_\_\_
3. Show that  $x^5 - 2x^2 + 7 = 0$  has atleast two imaginary roots.
4. If  $\omega$  is an imaginary cube root of unity, then the value of  $1 + \omega + \omega^2$  is \_\_\_\_\_
5. What is the value of  $\text{Arg } z$  for positive real axis,  $z = x$ ?



Answer any 8 questions from this part. Each question carries 2 marks. (8x2=16)

6. Show that the set of all integers is countable.
7. If  $\alpha, \beta, \gamma$  are the root of the equation  $ax^3 + bx^2 + cx + d = 0$ , then find the values of  $\alpha + \beta + \gamma$  and  $\alpha\beta\gamma$ .
8. Find the condition that the cubic equation  $x^3 - lx^2 + mx - n = 0$  should have its roots in arithmetical progression.
9. If  $\alpha, \beta, \gamma$  are the root of the equation  $8x^3 - 4x^2 + 6x - 1 = 0$ , find the equation whose roots are  $2\alpha + 1, 2\beta + 1, 2\gamma + 1$ .
10. State De Gua's rule.
11. What do you mean by reciprocal equation? Give an example.

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12. Describe the discriminant of the cubic equation  $ax^3 + 3bx^2 + 3cx + d = 0$ .
13. Transform  $x^3 - 6x^2 + 5x + 12 = 0$  into an equation lacking the second term.
14. If  $a, b, c$  are the roots of the cubic equation  $x^3 + px^2 + qx + r = 0$ , find the value of  $\frac{1}{a^2b^2} + \frac{1}{b^2c^2} + \frac{1}{c^2a^2}$ .
15. What are the imaginary cube root of unity?
16. Find the polar form of  $z = 1 + i$ .

## PART - C

Answer any 4 questions from this part. Each question carries 4 marks. (4×4=16)

17. If  $A$  is a set with  $m$  elements and  $B$  is a set with  $n$  elements and if  $A \cap B = \phi$ , then prove that  $A \cup B$  has  $m + n$  elements.  $h(t) = \begin{cases} f(t) & \text{if } t \leq m \\ g(t-m) & \text{if } t > m \end{cases}$
18. Solve the equation  $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ , given that the sum of the two of its roots is zero.
19. Find the rational roots of  $x^4 - 39x^2 + 46x - 168 = 0$ .
20. Solve  $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$ .
21. Describe the behaviour of roots of a cubic equation in terms of its discriminant.
22. Find the value of  $\sqrt{1+i}$ .
23. Find the fifth root of  $(-1)$ .

## PART - D

Answer any 2 questions from this part. Each question carries 6 marks. (2×6=12)

24. State and prove Cantor's theorem.
25. If  $\alpha, \beta, \gamma$  are the root of the equation  $ax^3 + 3bx^2 + 3cx + d = 0$ , then find the values of
  - a)  $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$
  - b)  $(\beta - \gamma)(\gamma - \alpha) + (\gamma - \alpha)(\alpha - \beta) + (\alpha - \beta)(\beta - \gamma)$ .
26. Find a real root of the  $x^3 + x^2 - 16x + 20 = 0$ .
27. If  $z_1$  and  $z_2$  are two complex numbers, prove that
  - a)  $|z_1 z_2| = |z_1| |z_2|$
  - b)  $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ .