K23U 2303

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023

(2019-2021 Admissions)

CORE COURSE IN MATHEMATICS

5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours | Max. Marks : 48

PART – A

Answer any 4 questions from this part. Each question carries 1 mark. (4×1=4)

1. Give example for a denumerable set.
2. If α, β, γ are the root of the equation $f(x) = 0$, then the equation whose roots are $-\alpha, -\beta, -\gamma$ is ______
3. Show that $x^5 - 2x^2 + 7 = 0$ has atleast two imaginary roots.
4. If ω is an imaginary cube root of unity, then the value of $1 + \omega + \omega^2$ is ______
5. What is the value of Arg z for positive real axis, $z = x$?

PART – B

Answer any 8 questions from this part. Each question carries 2 marks. (8×2=16)

6. Show that the set of all integers is countable.
7. If α, β, γ are the root of the equation $ax^3 + bx^2 + cx + d = 0$, then find the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$.
8. Find the condition that the cubic equation $x^3 - lx^2 + mx - n = 0$ should have its roots in arithmetical progression.
9. If α, β, γ are the root of the equation $8x^3 - 4x^2 + 6x - 1 = 0$, find the equation whose roots are $2\alpha + 1$, $2\beta + 1$, $2\gamma + 1$.
10. State De Gua's rule.
11. What do you mean by reciprocal equation ? Give an example.

P.T.O.

12. Describe the discriminant of the cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$.

13. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

14. If a, b, c are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, find the value of $\frac{1}{a^2b^2} + \frac{1}{b^2c^2} + \frac{1}{c^2a^2}$.

15. What are the imaginary cube root of unity ?

16. Find the polar form of $z = 1 + i$.

PART – C

Answer any 4 questions from this part. Each question carries 4 marks. (4x4=16)

17. If A is a set with m elements and B is a set with n elements and if $A \cap B = \phi$, then prove that $A \cup B$ has m + n elements.

18. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that the sum of the two of its roots is zero.

19. Find the rational roots of $x^4 - 39x^2 + 46x - 168 = 0$.

20. Solve $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$.

21. Describe the behaviour of roots of a cubic equation in terms of its discriminant.

22. Find the value of $\sqrt{1+i}$ [illegible]

23. Find the fifth root of (–1).

PART – D

Answer any 2 questions from this part. Each question carries 6 marks. (2x6=12)

24. State and prove Cantor's theorem.

25. If α, β, γ are the root of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, then find the values of

a) $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$

b) $(\beta - \gamma)(\gamma - \alpha) + (\gamma - \alpha)(\alpha - \beta) + (\alpha - \beta)(\beta - \gamma)$.

26. Find a real root of the $x^3 + x^2 - 16x + 20 = 0$.

27. If z_1 and z_2 are two complex numbers, prove that

a) $|z_1 z_2| = |z_1| |z_2|$

b) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$.